Notes on algorithmic research, aggregations in economic theory, and the unilateralism of the induction principle

Edgardo Bucciarelli[*] & Andrea Oliva[**]

[*] University of Chieti-Pescara (Italy), Dept. PPEQS, Section of Economics and Quantitative Methods
[**] Abruzzo Academy Foundation, United Nations Academic Impact, New York
[*] Corresponding author: edgardo.bucciarelli@unich.it

To be submitted to the first Zero/Minimal Intelligence (ZI/MI) Agents Conference - Yale School of Management and the Cowles Foundation for Research in Economics (New Heaven, CT, October 22–24, 2020)

Extended abstract

Drawing inspiration from Kurt Gödel's (1986) [1931] and Alfred Tarski's (1983) [1933] seminal contributions to modern mathematics and metamathematics, this paper is concerned with extending and integrating the understanding of the logical framework approach to aggregations in economic theory. By virtue of Gödel's legacy, Tarski further analysed and proved how metalogic can solve relatively simple mathematical questions that cannot be decided either on an axiomatic basis or by inference rules with which they are formally expressed. In the field of economics, however, very few efforts have been made to focus on the theory of natural numbers (founding the set of relative integers) that emerges—and is decidable—in terms of a metalogical abstraction of aggregations. Such an investigation promises to give valuable insights into the way logic and metalogic can reshape the concept of aggregations from micro to macroeconomic quantities. Indeed, relying on a purely constructivist and post-rationalist analysis of economics, the paper involves locating a point of focus and suggesting what can be included within that line of economic theory known as the aggregation problem or, simply, aggregations—both micro and macro—which serves, primarily, the logical purpose of generalisations in language within aggregation theory. After all, as is common knowledge, all scientific disciplines—with the exception of most of the political economy—describe the phenomena under investigation differentiating between a lower (i.e., micro) and a higher (i.e., macro) level, whereby the latter analyses the aggregate results and is normally not isomorphic to the former (e.g., see Simon & Ando, 1961; Anderson, 1972; Prigogine, 1980; Churchland & Sejnowski, 1995; and Kirman, 2016). From an epistemological and systemic perspective, however, the aggregate of interacting individual entities tends to the emergence of properties that cannot be reduced to the behaviour of individual, standard ideal, and representative components of the aggregate itself (e.g., see Kirman, 1992). The social sciences addressed the correspondence between micro and macro relationships since the beginning, placing this issue at the centre of the debate surrounding the aggregation problem and other controversies especially in the second half of the nineteenth century (e.g., see the famous Econometrica debate in the late 1940s). Some developments coming out of this debate helped pave the way for today's openness, acknowledging the interconnectedness of research methods and epistemologies, revealed so clearly as in the study of the relationship between human agents and the emergent, social aggregates to which they give shape, ecologically (e.g., see Schelling, 1978; Stephens & Krebs, 1986; and Miller & Page, 2007). If by its very nature the economic theory involves computable aggregations by grouping similarities, how therefore do economic aggregates emerge and how should these aggregates be associated with or represent the micro-variables from which they are originated? In our conception of economics as a science, aggregations in economic theory serve logical and metalogical purposes of cognitive generalisations before any other consideration involved in the above debate (e.g., in language and meta-logical aspects), being extremely useful for reasoning about macro-categories of entities cognitively and constructivistly viewed. However, the main point is that aggregations—as they were designed in standard economics—suffer from shortcomings and related shortcuts that need further investigations, whereby broad generalisations tend to lose accuracy in describing their basic epistemology, inner logic, and constraints of the process if they are not also accompanied by a sound foundational basis for doing so and any supporting evidence (see also Hartley, 1997; and Kirman, 2010). In particular, our point of focus is kind of specifically on this regard, resorting to a two-pronged methodological approach. The first approach is logical in nature and allows us to design a minimal-intelligence algorithm process flow based on biological properties of organic factors having, individually, a very meagre intelligence and working with no supervision (among others, see von Neumann, 1951; Gilbert, 1986; Kirman, 1993; Grosan & Abraham, 2006; and Chen, 2015). If we

consider, indeed, social science broad enough to encompass the 'social behaviours' of minimal-intelligence life forms, such as micro organic structures and virus-like agents that interact with macro living organisms or else emerge as a result of self-replication, it would be worth studying a minimal living entity/system capable of producing copies of itself as well as providing a reductionist theory of aggregations and the numerical algorithms—with its application. Particularly, we rely on microbiological entities through which the concept of aggregating minimal-intelligence agents can be build-up from simpler to more complex life forms by means of an inferential—i.e., computable—multi-staged algorithmic process as an aggregate. In other words, such an algorithm can be developed using an induction principle, which bases the mathematical truth of an induced consequence on the assumed truth of the logical antecedent or immediately preceding premise. Therefore, in a computable minimal-intelligence microbiological aggregate, the induction principle involves a mutual interaction of agents by means of recursive functions. It is worth noting that the advantage of providing this first (logical) approach lies in the fact that the algorithm qualifying these minimal-intelligence entities is explicitly of an inductive nature (inferential) and is expressed by a recursive form. Working along these lines, the second approach is metalogical in terms of abstraction characterising the aggregation process conceived algorithmically. The aggregation process, indeed, first describes the individual human agents through economic variables included in a function of a given form, then transforms this function into another formally identical to the previous one whose output is an expression of the average of the individual agents' utilities (e.g., see Lewbel, 1989): This is, *de facto*, a novel algorithm that can be represented in terms of a flowchart as a first approximation as exemplified in the paper. What is of specific relevance in this approach is resorting to metalogic which helps us demonstrate the impossibility to reverse an aggregation process; yet still, metalogic is a necessary but not sufficient condition for modelling aggregate states of economic behaviour in a purely mathematical sense. Nevertheless, wherein the logical approach, which returns the two-staged algorithm, and the metalogical approach, which allows us to demonstrate algorithmically the irreversibility of the aggregation process, are related to each other? Roughly speaking, both approaches are understood in terms of their common algorithmic nature as well as of the common nature of both processes as aggregates. In a nutshell, the logical and metalogical approaches lead to recognise the common algorithmic nature of both processes by virtue of the induction principle as pervading the automatism of both algorithmic processes in question. With this in mind, we cannot help but wonder—and this is the main research question—whether aggregations, thus the induction principle, can be reversed in order to return a consistent result in moving from micro to macroeconomics. Certainly, the answer to this question is negative. We demonstrate in the paper, indeed, that the irreversibility of aggregations, as a consequence of the irreversibility of the induction principle, brings out the analytical inconsistency of the standard economics methodological approach in making aggregations (see also de Finetti, 1959 [1943], 1975 [1970]; and Kirman, 1992). Methodologically, we prove why the induction principle is unilateral by acting in two steps: First, we consider any two inductive processes, each of them holds by itself axiomatically as stated by Peano (1889); then we prove the unilateralism of induction principle through the mutual induction of these processes each other. Mutual induction of any two inductive processes is intended as starting from any step of one of the two processes, where any step of the other process is obtained inductively, being both processes consecutively connected by induction. Consistent with the above demonstration of the unilateralism, the use of mutual induction represents a further novelty of this paper. In addressing this issue, it would be extremely helpful to recall that an inductive process tends to infinity; accordingly, to consecutively connect any two inductive processes, we assume that one process must be the mirror image of the other in the strict sense. Although this assumption ensures that two processes are consecutively connected, it implicitly violates the third axiom stated by Peano (1889). By and large, therefore, faced with such unilateralism of the induction principle, we point out that this unilateralism affects the consistency of aggregations in moving from micro to macroeconomics: It means that each aggregate quantity tends to return a partial description of 'micro' quantities it aspires to formalise in 'macro' or, in other words, between an aggregate and its micro foundations does not exist any isomorphism. As such, to the best of our knowledge, it seems our work is the first to point out that it might be strategic to reconsider aggregation processes purely by virtue of the unilateralism of the induction principle and, thus, the metalogical abstraction of aggregations.

Keywords: Logic in economics; Algorithmic social science research; Minimal-intelligence; Induction of processes; Mutual induction; Metalogical reasoning; Consistency of aggregating relations.