A comparison of zero and minimal intelligence agendas in Markov Chain voting models

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1. Introduction: Motivations

D.K. Gode and Shyam Sunder (1993- JPE google scholar -- cited by 2054, 1993 - Friedman & Rust "Double Auction Market" text, 1997 - QJE) established that markets populated by their **Zero Intelligence** (ZI) traders reach high efficiency and converge in price towards competitive equilibrium. ZIs bid **randomly** over [0,v] and or ask **randomly** over [c,H].

A wide range of strategies may be called "minimal intelligence" (MI).

2020: 1st ZI MI Conference:

> Do simple ZI / MI strategies offer insights into non-market environments?
>
> What about voting environments ?

1. Introduction: Methodology and Research Questions

Methodological Question: Many ZI/MI models are stochastic models and generators of Markov Chains. Typically we simulate these models. But instead, sometimes we can calculate empirical stationary distributions of these Markov Chains. Is this practical? What are the limits?

Assume: Collection of robots (or AIs) that have preferred points on a two-dimensional grid of points. Single robots vote truthfully and non-strategically. Agenda is set by one of two randomized rules: ZI or MI

Research Questions:

Does the theoretical literature in political science/voting
have anything to say about the outcomes when robots are voting?

Will the robots collectively pick a “reasonable” point or run amok?

If the robots cannot settle on a specific point, what can we say about the cloud or distribution of points?

Are some points more likely than others?

Voting Agendas are well known to be crucial to outcomes. If the agenda is either:

(ZI) completely random; or

(MI) constrained to locally winning proposals (plus the status quo),

What effect does the agenda affect the outcome?

1. Introduction: Methodology and Research Summary

Methodological Question: Many ZI/MI models are generators of Markov Chains…. Are MC calculations practical?

Yes. Stationary probability distributions can be calculated for Markov Chains and are within reach of practical GPU-based computing (python; Google Co-Lab). These calculated stationary distributions are then used to study the research questions.

Assume: Collection of robots (or AIs) that have preferred points on a two-dimensional grid of points. Single robots vote truthfully and non-strategically. Agenda is set by one of two randomized rules: ZI or MI

Research Questions with summarized results:

Does the theoretical literature in political science/voting
have anything to say about the outcomes when robots are voting? **YES.**

Will the robots collectively pick a “reasonable” point or run amok? **It depends; “run amok” is possible.**

If the robots cannot settle on a specific grid point, what can be said about the cloud or distribution of points? **Depends on voting environment and agenda.** Are some points more likely than others? **YES**

Does the agenda (ZI or MI) affect the outcome? **YES. MI tends to have higher variance.**

The “smarter” agenda (MI) can yield worse outcomes (than ZI). Perhaps bumbling can add stability.

1. Introduction: Literature Review: Condorcet (1780s)

Marquis de Condorcet (1780s)

Jury Theorem:

With regard to binary decisions (yes or no / guilty or not-guilty) -- making a group decision via majority rule lowers the error rate (vs. individual decision).

Voting Paradox:

Pairwise votes for 3 voters among 3 items can cycle. A>B B>C and C>A

Examined further in preprint section 4.

1. Introduction: Literature Review: Black (1948)

Spatial Voting in 1 Dimension

Black's Theorem

N-odd Voters each have a preferred point on a line and distance-based preferences.

=> median voter's preferred point beats any other point in a pairwise vote

Example: Number of voters = 7 (D is median)

A B C **D** E F G

1. Introduction: Literature Review: Plott(1967)

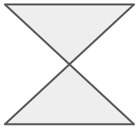
Spatial Voting in 2 or more dimensions

For N>1 dimensions, Black's theorem does **not** hold.

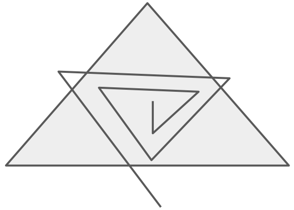
A similar equilibrium is only possible with specific arrangements of voter ideal points…

Plott (1967) conditions for equilibrium

Odd number of voters. At some voter's ideal point, the gradients of all other voter's utility functions must align in pairs and in direct opposition to each other.



In the absence of equilibrium there is a cycle.



1. Introduction: Literature Review: McKelvey, Ferejohn et.al., Schofield (1970s)

For N>=2 dimensions, when the Plott conditions are not satisfied:

- An agenda of pairwise votes exists that connects any two policy points.
- A space-filling cycle exists
- If the agenda is suitably random, one can use Markov-chain techniques to place bounds on the probability that the voting system reaches a specific distance from the center.

2. Models: Model Arena

Some finite set **F** of feasible alternatives; An initial status quo f[0]

For each t=0,1,2,3,4,...

1. An agenda maps from f[t] to a randomly-selected challenger c[t]
2. Agents vote (truthfully, non-strategically) on f[t] vs. c[t]
3. The winner becomes f[t+1];
   Iterate by advancing time to t+1 and repeating the process at step 1.

2. Models: Zero Intelligence Agenda

ZI Agenda:

Pick a challenger uniformly at random from the feasible set **F**

Why is this “zero intelligence”?

Requires no knowledge of agent preferences or voting history

2. Models: Minimal Intelligence Agenda

MI Agenda:

For the current choice f, construct **W(**f**)** = strict voting winners vs f.

Choose a challenger policy uniformly at random from { f ∪ **W(**f**)** }

Why is this “minimal intelligence”?

Requires knowledge of majorities’ preferences, so not zero intelligence.

Not history dependent.

Does not require thorough analysis of each possible winning choice.

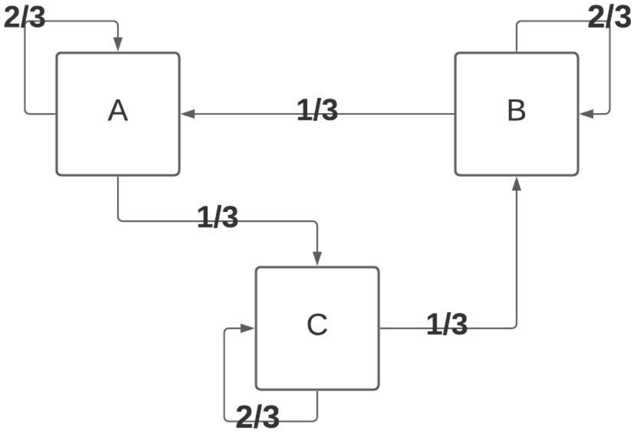
Instead it randomizes.

3. Markov Chain Methodology: What is a Markov Chain?

A Markov Chain is a sequence of random variables defined by:

- A set of states
- An initial state or probability distribution over states
- A matrix P(i,j) of transition probabilities between states;
  Where P(i,j) evolves a state probability vector π through a left vector-matrix multiplication $\pi[t+1] = \pi[t]*\mathbf{P}$

$$\begin{bmatrix} 2/3 & 0 & 1/3 \\ 1/3 & 2/3 & 0 \\ 0 & 1/3 & 2/3 \end{bmatrix}$$



3. Markov Chain Methodology: Procedure

1. Identify the Markov Chain for a voting environment and agenda (ZI, MI)
2. Check diagonal of P for absorption points (in our examples, none)
3. Assume a cycle and find the stationary distribution π^*
    a. Method 1: Raise P to a high power. All rows should converge toward the stationary distribution π^*
    b. Method 2: Solve a set of linear equations for the stationary distribution π^*
4. Examine properties of the stationary distribution π^*

For different voting agendas in the same environment, we may see differences in:

a. Distribution Shapes
b. Variance
c. “Fairness”

3. Markov Chain Methodology: Diagnostics and Limits

We examined several diagnostics and obtain values less than 10^-10

- Check that stationary distribution is stationary: $\|\pi^* P - \pi^*\|$ is small
- Check that stationary distribution sums to 1.0
- Compare || method 1 - method 2 || solutions for stationary distribution

Limits:

- GPU-based calculation recommended. Python has GPU matrix libraries.
- Google CoLab Python is limited primarily by GPU memory on the server
- GPU memory is exhausted at around a [-80,80] x [-80,80] grid of alternatives

  => 25,921 x 25,921 transition matrix (~ 671 million values)

3. Markov Chain Diagnostic Tables (from paper)

Table 1 Spatial Voting: Diagnostics for calculation of π^*

	grid	agenda	$\|\|\mathbf{F}\|\|$	power	$\|\|\pi - \pi P\|\|_1$	$\sum \pi - 1$	$\|\|\pi_{\text{method 1}} - \pi_{\text{method 2}}\|\|_1$
0	80	ZI	25921	4096	2.861e-15	-1.386e-11	NA
1	80	MI	25921	64	1.752e-14	-1.987e-14	NA
2	60	ZI	14641	4096	9.399e-16	-3.735e-12	3.783e-12
3	60	MI	14641	64	8.549e-15	-5.551e-16	2.698e-14
4	40	ZI	6561	1024	1.113e-14	-6.815e-13	7.59e-13
5	40	MI	6561	64	3.663e-15	0	6.942e-15
6	20	ZI	1681	256	3.072e-14	-1.109e-13	2.967e-13
7	20	MI	1681	32	4.892e-16	-2.22e-16	2.358e-15

Table 2 Budget Voting: Diagnostics for calculation of π^*

	agenda	$\|\|\mathbf{F}\|\|$	power	$\|\|\pi - \pi P\|\|_1$	$\sum \pi - 1$	$\|\|\pi_{\text{method 1}} - \pi_{\text{method 2}}\|\|_1$
0	ZI	5151	64	1.35e-14	-2.56e-13	2.56e-13
1	MI	5151	32	1.75e-15	2.22e-16	2.53e-15

4. Example: Condorcet Cycle (preprint; p.19)

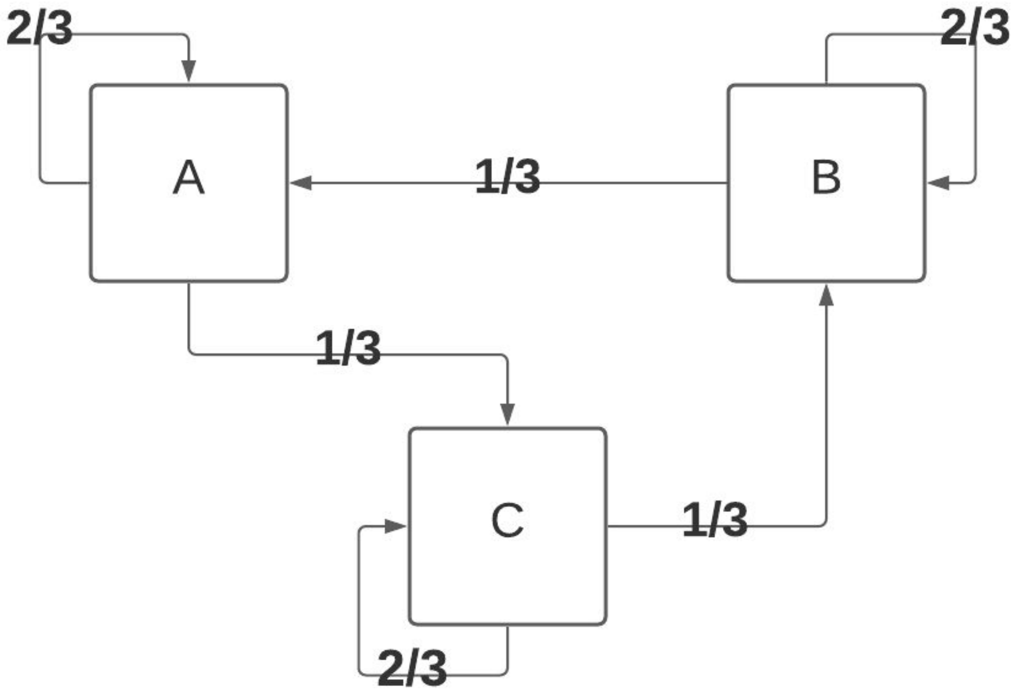
Table 3 Utility functions for a Condorcet Cycle

Voter	A	B	C
1	3	2	1
2	1	3	2
3	2	1	3

The voter utility profile in Table 3 is well known to lead to an intransitive majority rule result $A > B; B > C; C > A$. This follows from examining each two-voter coalition: Voters 1 and 3 prefer A to B. Voters 1 and 2 refer B to C. But voters 2 and 3 prefer C to A.

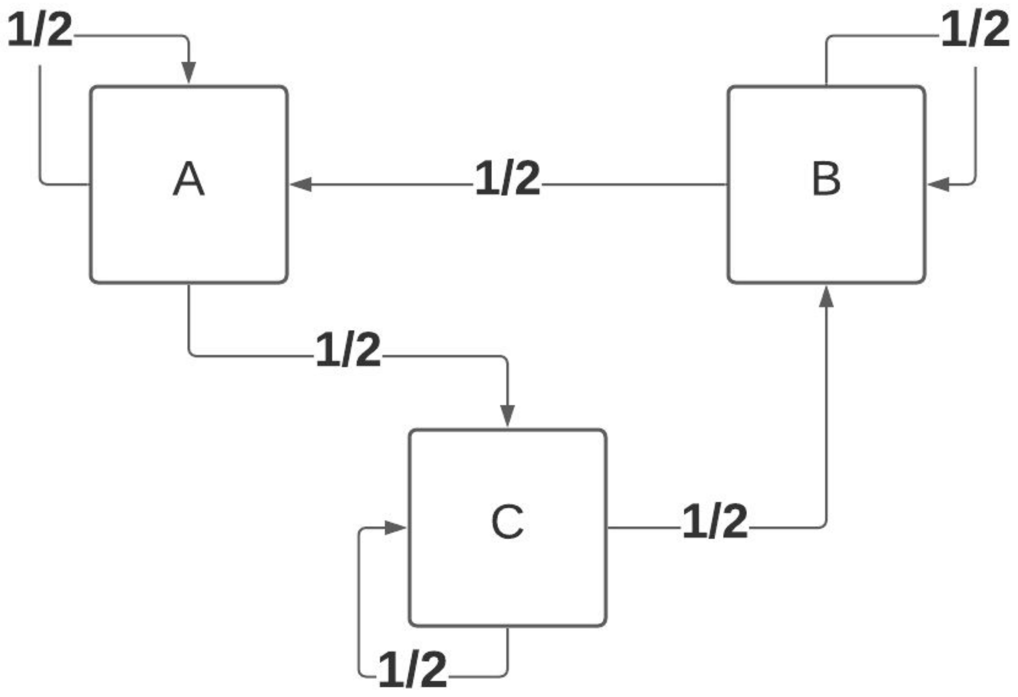
4. Condorcet Cycle: ZI Agenda Markov Chain

$$\begin{bmatrix} 2/3 & 0 & 1/3 \\ 1/3 & 2/3 & 0 \\ 0 & 1/3 & 2/3 \end{bmatrix}$$



4. Condorcet Cycle: MI Agenda Markov Chain

$$\begin{bmatrix} 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$



4. Condorcet Cycle: Time Evolution (t=4)

$$P_{\mathrm{ZI}}^{4} = \begin{bmatrix} 0.296 & 0.296 & 0.407 \\ 0.407 & 0.296 & 0.296 \\ 0.296 & 0.407 & 0.296 \end{bmatrix}$$

$$P_{\mathrm{MI}}^{4} = \begin{bmatrix} 0.3125 & 0.3750 & 0.3125 \\ 0.3125 & 0.3125 & 0.3750 \\ 0.3750 & 0.3125 & 0.3125 \end{bmatrix}$$

4. Condorcet Cycle: Stationary Distribution is [⅓,⅓,⅓]

Guess [⅓,⅓,⅓] ?

$$[\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3}] * \begin{bmatrix} 2/3 & 0 & 1/3 \\ 1/3 & 2/3 & 0 \\ 0 & 1/3 & 2/3 \end{bmatrix}$$

= [⅓, ⅓, ⅓]

Guess [⅓,⅓,⅓] ?

$$[\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3}] * \begin{bmatrix} 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$

= [⅓, ⅓, ⅓]

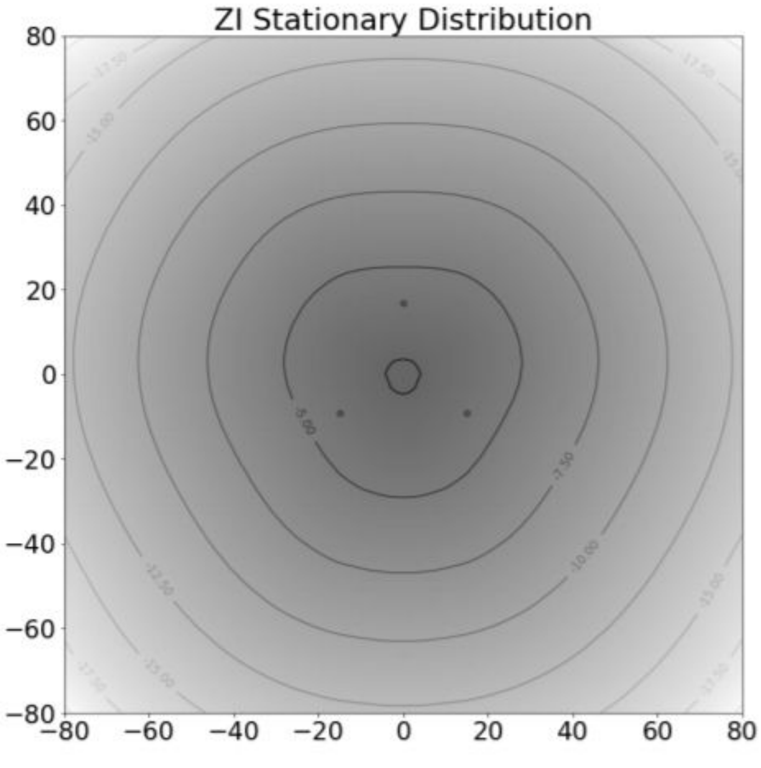
4. Condocet Cycle Example: Conclusions

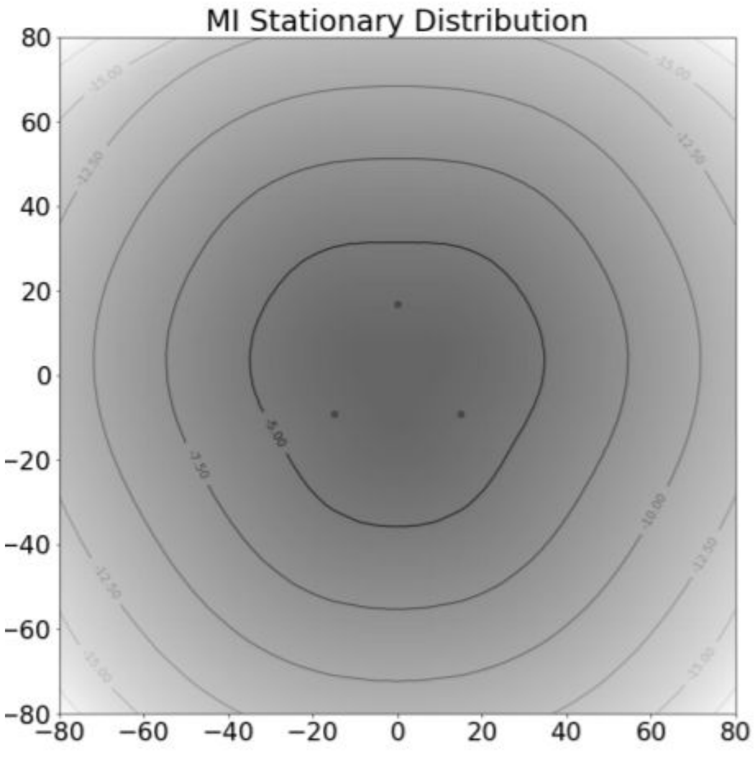
- Markov chain for MI agenda evolves faster than Markov Chain for ZI agenda
- Both have same stationary distribution, equal probability of each outcome

5. Example: Spatial Voting with Triangle of Ideal Points (3 voters; square grid)

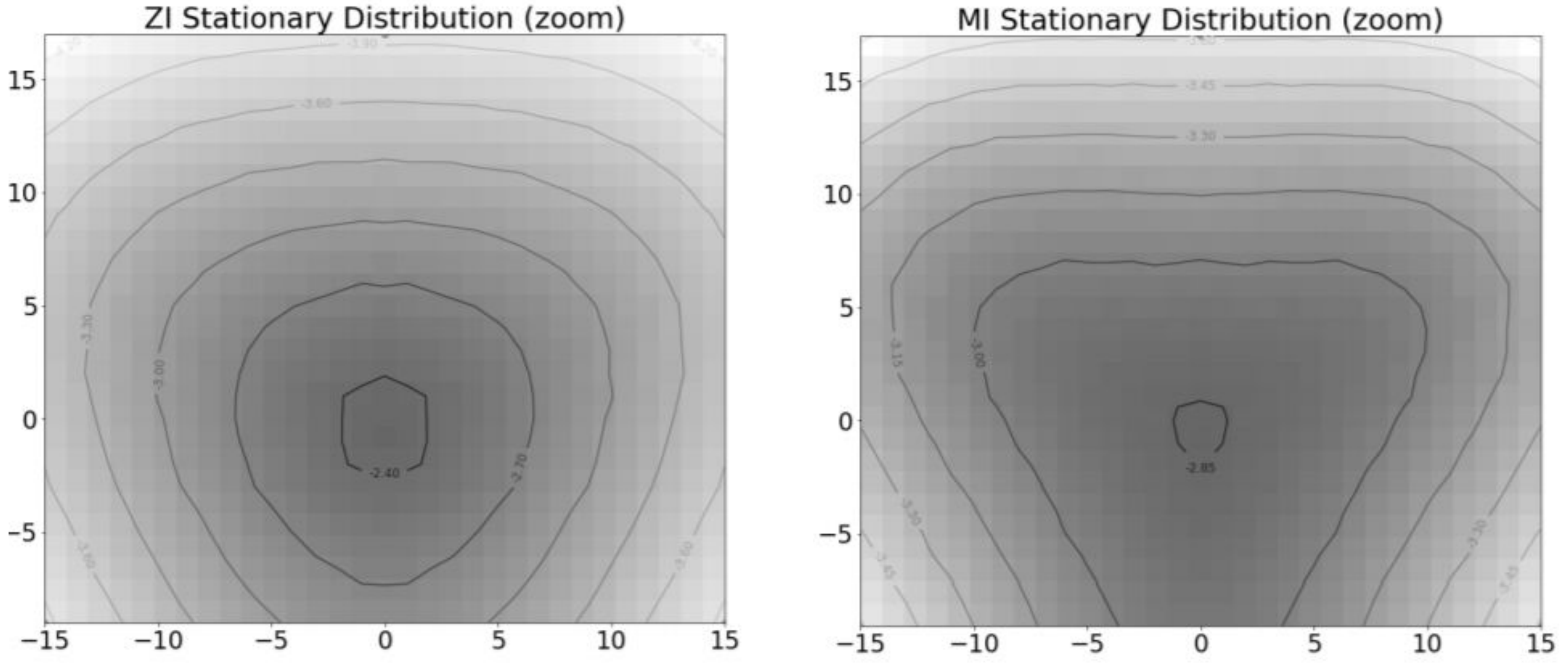
- 2D (X,Y) integer grid; 3 voters
- X, Y integers ranging from -N to +N (treatments of N: 20,40,60,80)
- Voter's utility functions are distance based from an individual ideal point.
- Voter 1 prefers (X=-15, Y=-9)
- Voter 2 prefers (X= 0; Y=+17)
- Voter 3 prefers (X=+15, Y=-9)
- Voters 1,2,3 preferred points form a near equilateral triangle
  - Side length is either 30 or sqrt(901)~30.02
  - Centroid is (0, -⅓)

5. Spatial Voting: ZI vs MI stationary distribution





5. Spatial Voting: ZI vs MI stationary distribution (zoom)



5. Spatial Voting: Stationary Dist Mean and Variance

Table 5 Spatial Voting: Long-term Means and SDs (values below 10^{-16} truncated)

	grid	agenda	$\overline{x} \pm SD(x)$	$\overline{y} \pm SD(y)$
0	80	ZI	0 ± 7.425	-0.3431 ± 7.424
1	80	MI	0 ± 10.82	-0.342 ± 10.81
2	60	ZI	0 ± 7.425	-0.3431 ± 7.424
3	60	MI	0 ± 10.82	-0.3419 ± 10.81
4	40	ZI	0 ± 7.421	-0.3428 ± 7.424
5	40	MI	0 ± 10.78	-0.3373 ± 10.77
6	20	ZI	0 ± 6.985	-0.2937 ± 7.123
7	20	MI	0 ± 9.032	-0.1452 ± 9.023

5. Spatial voting: Properties of stationary distributions

Table 6 Spatial Voting: Summary

	grid	agenda	$p_{\text{within triangle}}$	$p_{\text{grid boundary}}$	p_{min} corners	p_{max} (0,-1)	H (bits)
0	80	ZI	0.6746	1.635e-11	1.329e-21	0.004945	9.815
1	80	MI	0.396	6.536e-10	6.785e-20	0.001467	10.93
2	60	ZI	0.6746	2.818e-08	5.549e-16	0.004945	9.815
3	60	MI	0.396	6.333e-07	1.596e-14	0.001467	10.93
4	40	ZI	0.6746	2.551e-05	1.821e-10	0.004945	9.815
5	40	MI	0.3964	0.0002538	2.32e-09	0.001468	10.92
6	20	ZI	0.6802	0.008638	4.664e-06	0.004944	9.679
7	20	MI	0.4579	0.02415	1.694e-05	0.001711	10.32

5. Spatial Voting: Conclusions

- Stationary Distribution for ZI agenda has (vs. MI):
  - Lower variance
  - Higher probability within ideal point triangle (pareto set) (~67% ZI vs ~%40 MI)
  - Lower probabilities at grid boundaries
  - Higher probabilities at center
- MI agenda Markov Chain has faster convergence

6. Example: Budget Voting (3 voters; split $100)

[X=0...100; Y = 0...100] Integer Grid restricted to lower triangle

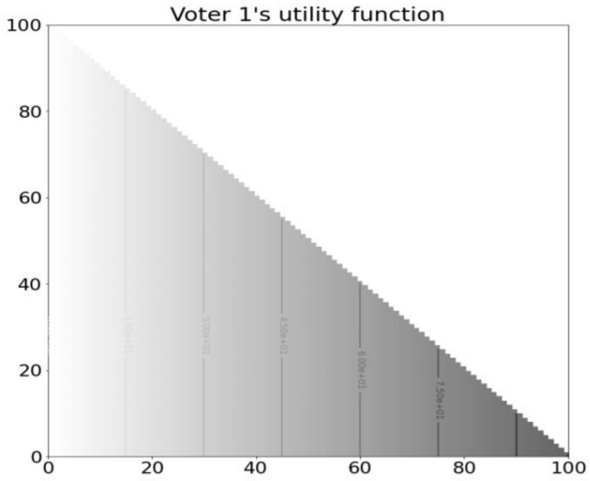
Voter 1 receives X dollars

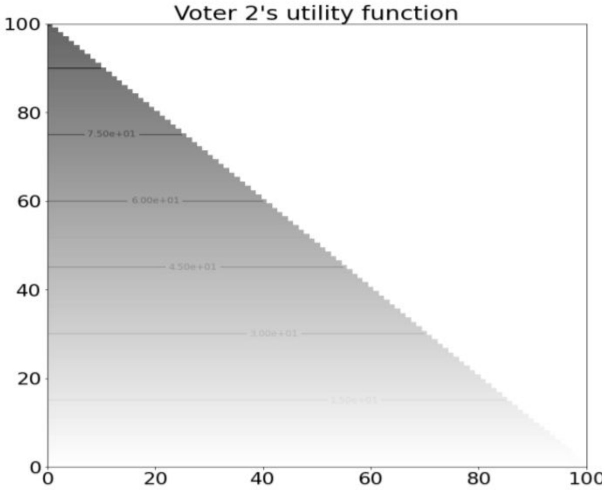
Voter 2 receives Y dollars

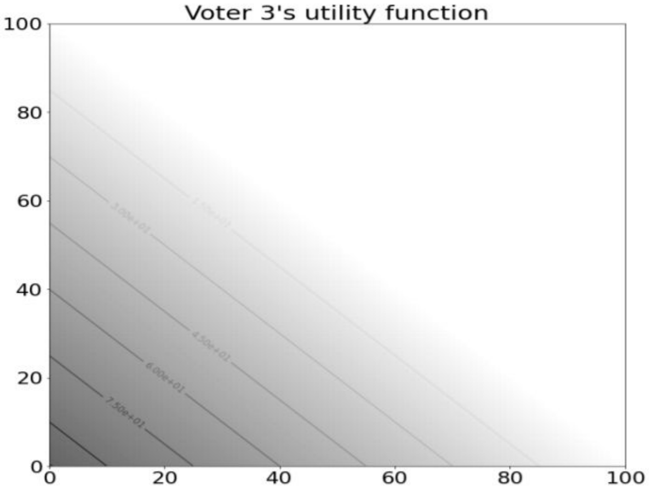
Voter 3 receives remainder = 100-X-Y dollars

In a robot or AI environment, the resource might be some other unit (not dollars) that is rivalrous and indivisible, such as energy consumption, CPU time, communications bandwidth, etc...

6. Budget Voting: Utility Functions







Cycling is possible: every state can be beaten
Unlike spatial voting example:
- Every state is in the Pareto set
- utility functions are not circular shaped

6. Budget Voting: Stationary Distributions -- ZI, MI agendas

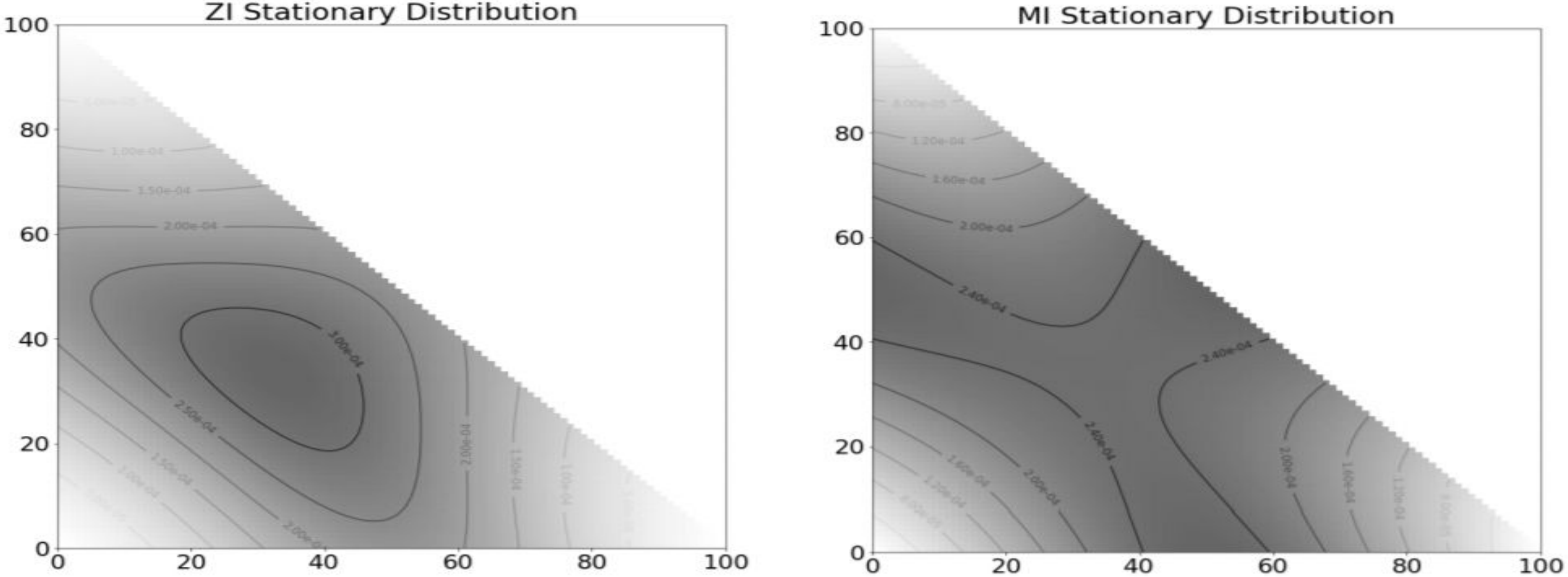
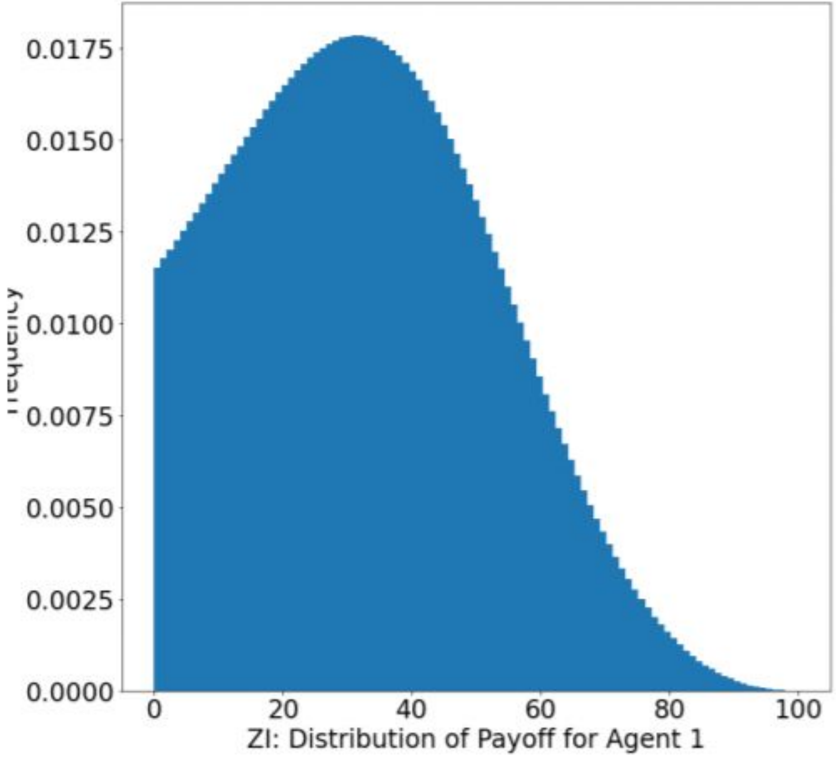
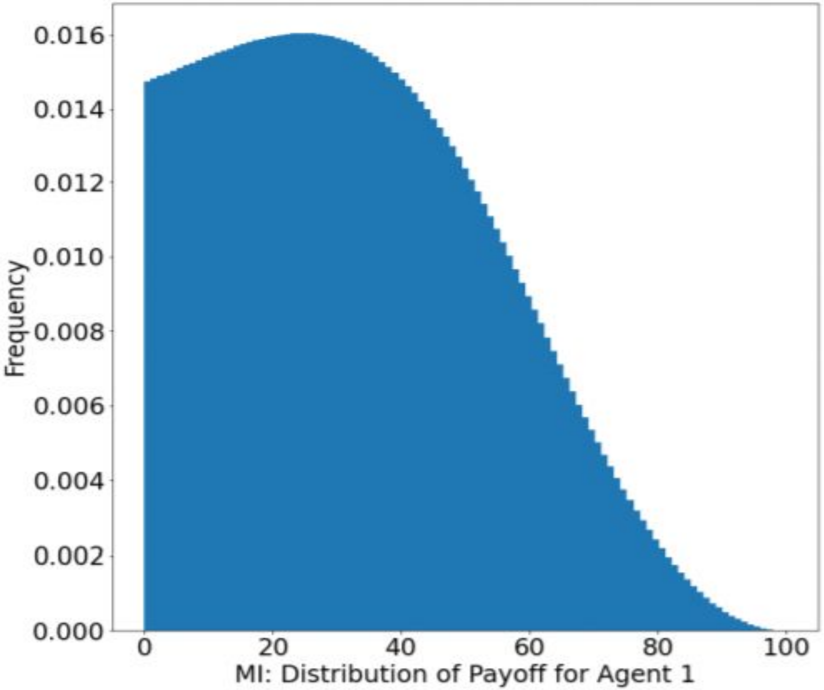


Fig. 8 Markov Chain stationary distributions for budget voting. Note that π^*_{ZI} has maxima at $(33,33),(33,34),(34,33)$ whereas π^*_{MI} has a saddle point near $(33,33)$ and constrained maxima at $(0,50),(50,50),and(50,0)$

6. Budget voting: Payoff distribution comparison





6. Budget voting: conclusions

ZI vs MI agenda

- Long-term Mean allocation ($\frac{1}{3},\frac{1}{3},\frac{1}{3}$) unaffected by ZI vs. MI agenda; variance is higher under MI agenda
- Shape of stationary distribution equal probability contours and maximum probability point moves from near equal allocation (ZI) to 50-50-0 allocation (MI)
- Probability of an agent receiving zero is lower with ZI agenda (3.4%) than with MI agenda (4.4%)

7. Conclusions: “Smarter” MI agenda worse than “ZI”

- Extreme outcomes (occur more frequently under MI agenda)
- Variance (higher for MI agenda)
- Pareto-set outcomes (less frequently under MI agenda)
- Shape of stationary distribution (changes under MI agenda)

7. Conclusions: Future work

- Other shapes and scenarios; comparison with previous human-subject experiment data. The McKelvey and Ordeshook (1990) review contains a number of configurations that could be investigated.
- Strategic Agents? Review of more recent work on spatial voting, especially in relation to Markov Chain voting models with strategic agents, may yield a more complete picture of what happens as “intelligence” increases. Kalandrakis and Penn’s research may have these pieces.
- Larger numbers of Agents
- More complex voting rules / institutions