IEL and ZI in the CDA

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The Continuous Double Auction (CDA)

A continuous, centralized market in which buyers and sellers are free at any time to make bids and asks and to accept the bids and asks of others. It is the preferred exchange mechanism of financial markets around the world.

Trade behavior and market performance in a controlled double auction setting were first studied by Smith (1962). Since then there have been thousands (millions?) of other CDA experiments.

In a one commodity market, prices and allocations converge to a competitive equilibrium with replication across periods. But that equilibrium is not reached *in the first period*.

There is a theory for the convergence with replication.

Easley-Ledyard (1993) supported by Lin et al. (2020)

Surprisingly after all this time, there is no generally accepted theory of price formation in the first period,

– although ZI seems prominent.

The first period is important because:

– It is the stylized situation of the immediate time after an unanticipated shift in demand-supply conditions.

– It is important to discover how price discovery occurs in this information sparse, disequilibrium situation.

We study the first period in experimental CDA markets.

Cason-Friedman (1996) tested three theories: Bayes Equilibrium (Wilson 87), BGAN (Friedman 91), and ZI (Gode-Sunder 93).

– "ZI now seems the only natural source for a null hypothesis in assessing the performance of any more complicated mechanism."

– ''None of these three models adequately explains price formation in double auction markets."

Lin et al. (2020) tested the same three theories using a data set of more than 9000 observations.

– "We find much stronger support for ZI theory compared to Cason-Friedman. It appears ZI explains dynamics within periods.

– "However, a non-negligible portion of our data falls outside of the simulated 95% confidence region."

Today, we consider

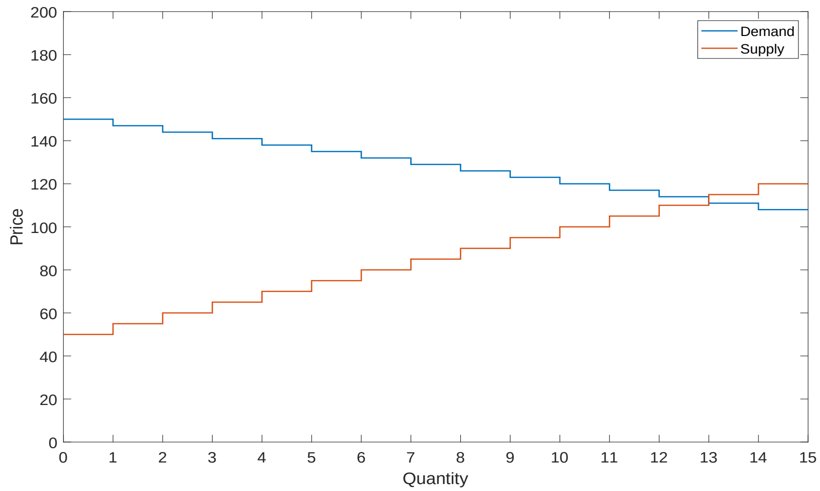
Two theories (involving simulation):
– ZI (Gode-Sunder): for obvious reasons
– IEL (Arifovic-Ledyard): has worked well in many synchronous move, repeated games (public goods, call market, etc). Will it work in the asynchronous, continuous game of the CDA?

– Two data sets: MobLab (the set used by Lin et al.) and SFU (a set we created for reasons detailed later)

We use a common demand/supply environment for both simulations and experiments.

The environment

5 agents on each side of the market. Each can buy/sell up to 3 units. No resale or buy back.



Equilibrium (price, quantity) = ([110,114], 13).

Note the asymmetry.

The ZI Model (Gode-Sunder)

Randomly pick a buyer or seller with replacement.

Buyer bids uniformly from [0,V], seller asks uniformly from [C,M].
– V is the value of the next unit the buyer is to buy and C is the cost of the next unit the seller is to sell. M is the maximum possible value.

A higher bid/lower ask replaces the trader's current outstanding bid/ask in the book.

When a bid/ask is higher/lower than the outstanding ask/bid, a transaction occurs at the earliest of that bid and ask.

The process continues for a fixed number of computer iterations.
– One option: "enough draws to allow sufficient time for trading"

The IEL Model in CDA (Arifovic-Ledyard)

Identical to ZI with the exception of *What to Bid*.

Each agent maintains a finite set of *considered bids*, $\mathcal{B}$.

Each considered bid is evaluated by asking what the payoff would have been at the last decision– called foregone utility W.

$\mathcal{B}$ and W are updated.
– Experimentation: randomly replaces some considered bids in $\mathcal{B}$ with others.
– Replication: a pairwise fitness tournament eliminates considered bids that would not have done well under W.

A bid is randomly selected from $\mathcal{B}$ in proportion to W.

IEL Bidding - Simplified

Let O^* be the best offer in the book.

If $V < O^*$,
– the buyer bids randomly from [0,V]. Same as ZI.

If $V \geq O^*$,
– if it is her first time to order, the buyer bids randomly from [0,V]. Accepts with probability $\left[\frac{V-O^*}{V-0}\right]$. Same as ZI.
– At each new opportunity, after not accepting, the buyer increases the probability that the bid comes from $[O^*, V]$.
IEL is less patient than ZI.

IEL learns to be more aggressive. Both are myopic (non-strategic).

IEL Simulations

Each run simulated the first period of an experimental session. There were 10,000 runs for each simulation.

We used the same parameters we have used in our previous work: $|\mathcal{B}| = 100$ and the probability any bid in $|\mathcal{B}|$ is selected for experimentation is $\mu = 0.033$. We always check the sensitivity of the results to the parameters.

Our long term goal is to have a calibration free model of the experimental lab.

Our results are robust to values for $|\mathcal{B}| \in [50, 500]$ and for $\mu \in [0.0033, 0.25]$.

IEL Simulation Results

draws	E	Q	ρ_B	P	BPS
20	48.5	4.5	0.37	103.9	0.49
30	63.8	6.4	0.44	104.4	0.48
40	74.1	7.8	0.48	104.8	0.48
50	80.3	8.8	0.5	105.1	0.48
60	84.5	9.5	0.52	105.4	0.48
70	87.0	10.0	0.53	105.7	0.48
80	88.6	10.3	0.53	105.9	0.48
90	89.7	10.5	0.54	105.8	0.48
100	90.4	10.7	0.54	106.1	0.48
110	90.9	10.8	0.54	106.0	0.48
120	91.2	10.8	0.55	106.1	0.48

E = efficiency, Q = quantity, ρ_B = buyers' correlation coefficient, P = price, BPS = buyers' profit split

Observations

- Efficiency and Price increase at decreasing rate in the number of draws.

  – (a) Trading is Marshallian (higher values and lower costs trade sooner on average)
  – (b) a Buyer and a Seller split the surplus on average.

- Prices converge to a Competitive Equilibrium (CE) from below.

  – (a), (b), and the asymmetry of equilibrium surplus
  – the supply curve is further below CE than the demand curve is above.

ZI Simulation Results

draws	E	Q	ρ_B	P	BPS
30	32.3	2.8	0.26	108.8	0.43
40	40.5	3.6	0.31	109.1	0.42
50	47.7	4.3	0.34	109.4	0.42
60	53.7	5.0	0.39	109.5	0.42
70	58.9	5.6	0.41	109.3	0.42
80	63.3	6.1	0.43	109.5	0.42
90	67.3	6.6	0.45	109.6	0.42
100	70.6	7.0	0.47	109.6	0.42
110	73.3	7.4	0.48	109.6	0.42
120	76.1	7.8	0.50	109.6	0.42
130	78.1	8.1	0.51	109.7	0.42

E = efficiency, Q = quantity, ρ_B = buyers' correlation coefficient, P = price, BPS = buyers' profit split

Observations

- Efficiency increases in the number of draws. But does not level off until about 700 draws

- Prices increase very slightly in the number of draws and converge to a CE from below.
  – The buyers' profit split is just about equal to the split in a competitive equilibrium.

- For each number of draws,
  – ZI efficiency is lower than IEL efficiency
  – ZI prices are higher than IEL prices.

Testing IEL and ZI with the MobLab Data Set (Lin et al. 2020)

Disclosure: Ledyard is on the Board of MobLab

The good news: This is a large (over 9000 markets), publicly available data set for CDA experiments. There are many replications across widely different worldwide subject pools.

There are 2090 observations with our configuration. We use only the first period from these.

The bad news: These are uncontrolled, classroom experiments with inexperienced subjects. Unlikely to be paid. Many violations of Individual Rationality

– 25% of traders have at least 5 of IR violations

– 4.3% of markets have an *average price* greater than the largest possible buyer value.

We will deal with this later.

Some Statistics

Efficiency	85.9
Quantity	11
Sellers' Correlation	0.57
Average Price	113.2
Buyers' Profit Split	0.36
Orders per period	55

How can we use these data to test IEL and ZI?

Approach #1: Calibrate the number of draws to these statistics.

For IEL: 65 draws gives 85.7 efficiency, 160 draws gives the quantity traded, and 80 draws gives the sellers' correlation correct. Which to use?

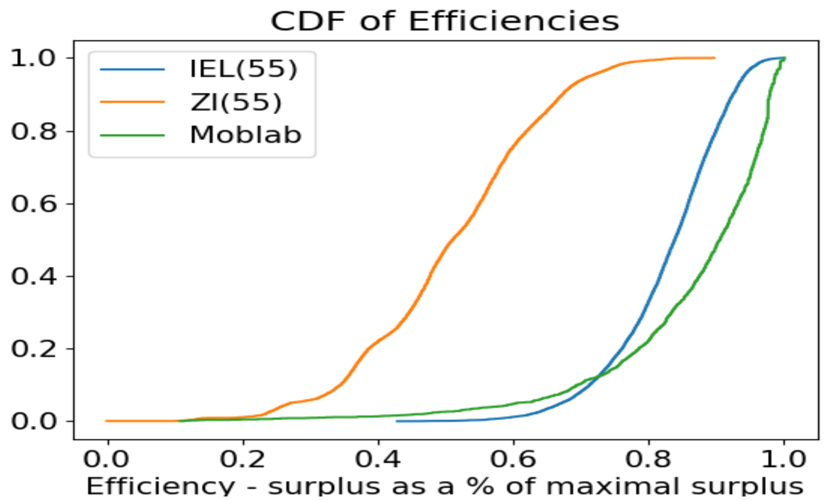
IEL vs. ZI vs MobLab

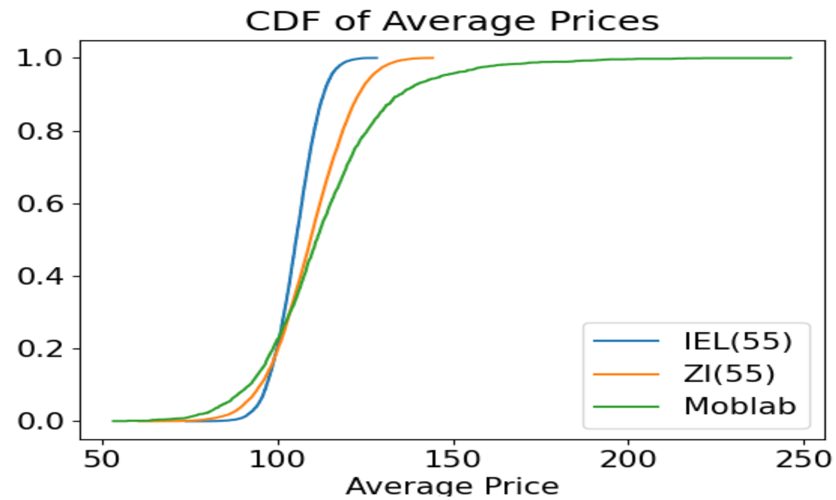
Approach # 2: (Our choice) Let the number of draws equal the average number of orders. This avoids calibration and *ex post* fitting. It ties our hands.

	MobLab	IEL (55 draws)	ZI (55 draws)
Efficiency	85.9	82.7	50.8
Quantity	11	9.2	4.7
Buyers' Correlation	0.56	0.51	0.37
Sellers' Correlation	0.57	0.53	0.36
Average Price	113.2	105.3	109.4
Buyers' Profit Split	0.36	0.48	0.43
Orders	55	55	55

IEL is much closer in efficiency. ZI is closer in price.

IEL vs. ZI vs MobLab





IEL is much better at explaining efficiency than ZI

– But, there are more high MobLab efficiencies than ZI or IEL.

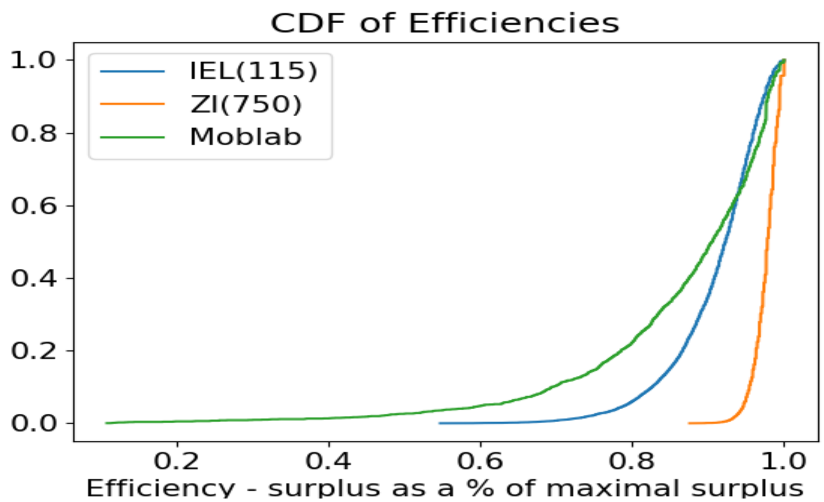
ZI is better at explaining prices than IEL

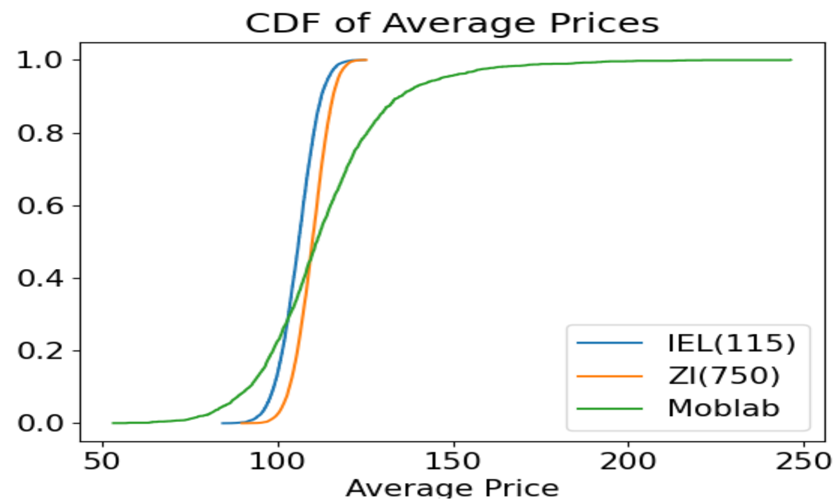
– But, 25% of MobLab prices are greater than the max ZI price.

Neither is a particularly good fit.

IEL vs. ZI vs MobLab – Full Trading

Have we inappropriately handicapped ZI by not allowing "enough draws to allow sufficient time for trading"?





No. Full trading is a worse fit for both IEL and ZI – too efficient, not enough average price dispersion.

We either need a better theory or a better data set.

We begin with the theory.

A better theory needs to explain the high level of IR violations.

– Neither ZI nor IEL allow such violations. This is consistent with standard economic theory – not using dominated strategies.

Rather than change IEL, we introduce heterogeneity into our set of agents.

Better Theory: Adding NI Agents to the Subject Pool

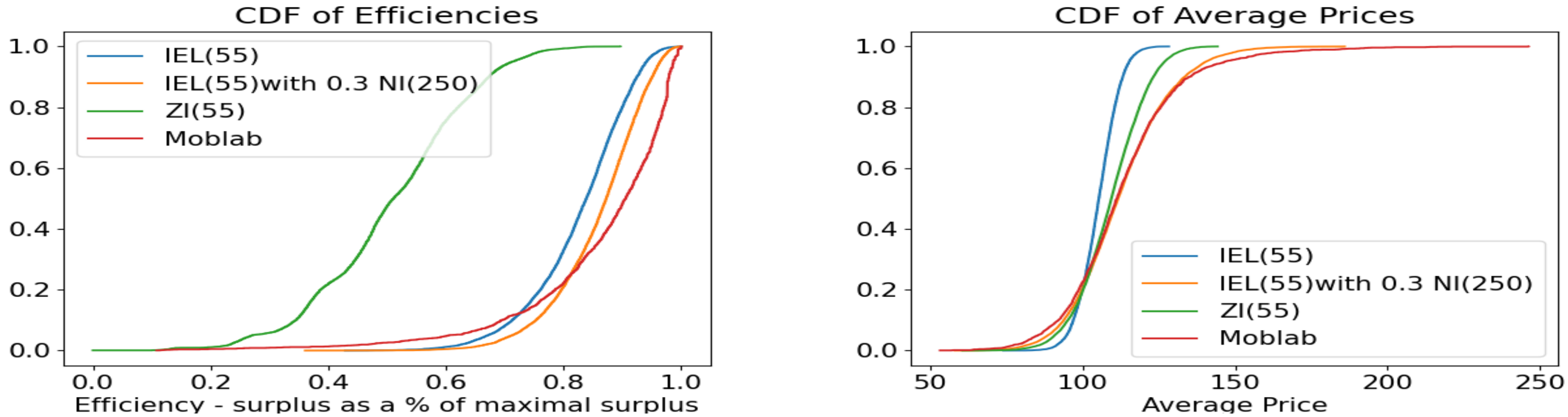
A No Intelligence (NI) agent randomly bids or asks from $[0, s_N]$ no matter what their values or costs.

– If s_N is the same as the upper bound for bidding, then an NI trader is the same as a ZI-U (ZI unconstrained) agent of Gode-Sunder (1993).

At the beginning of each run, we randomly assign traders to be either NI, with probability 0.3, or IEL, with probability 0.7. All else remains the same.

A grid search determined that $\eta = 30\%$ and $s_N = 250$ gave a best fit measured by the mean squared error to the MobLab average efficiency and price. (The upper bound on biding was 300.)

IEL(55) vs. 70%IEL(55) + 30%NI(250) vs. ZI(55) v MobLab



IEL+NI is better than both IEL and ZI at explaining both efficiency and price than both IEL and ZI.

Better Data: Our own experiments - SFU

We ran three sessions on-line with the SFU subject pool. We used the Flexemarkets interface.

Each session involved 12 periods of CDA trading: 2 practice, then 10 regular.

Same demand-supply environment. To create the conditions such that subjects would believe each period was new:
Each period
– buyers (sellers) were randomly distributed on the demand (supply) curve
– a random amount was added to all values and costs.

Traders were informed that all values and costs were drawn from $[0, 200]$. Each trader knew only their own value/cost.

SFU v MobLab

	SFU	MobLab
Efficiency	71	85.9
Quantity	7.6	11
Buyers' Correlation	0.25	0.56
Average Price	105.7	113.2
Buyers' Profit Split	0.47	0.36
Orders	40	55
Significant IR violations	4%	25%

The controlled experiments yield different results from the classroom experiments.

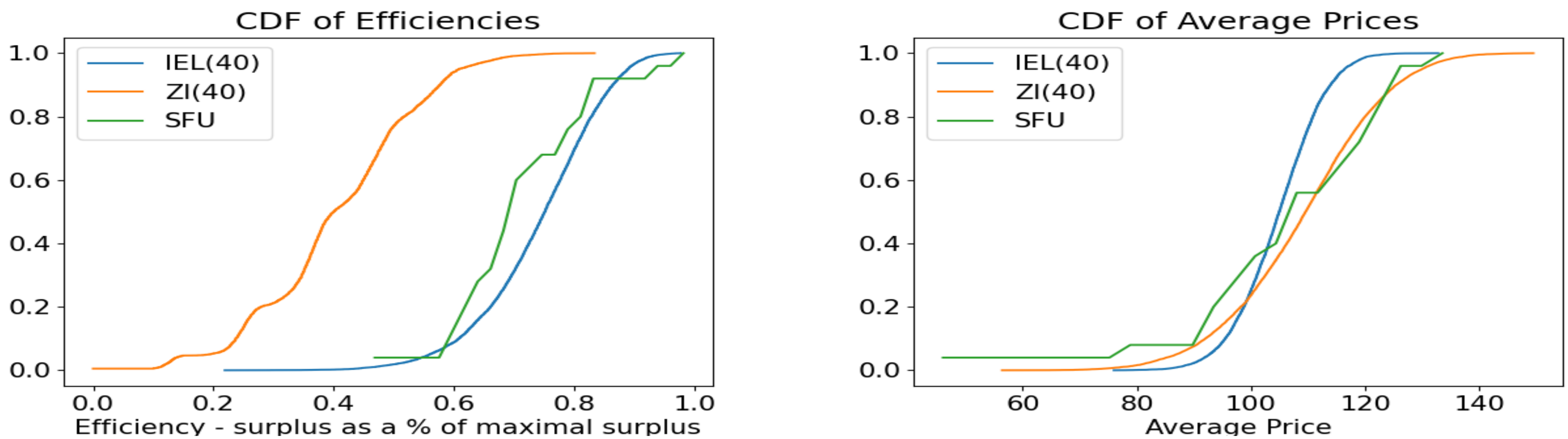
– There is less trading in SFU which implies lower efficiencies

– There is a more equal split of profits in SFU which implies lower average prices.

– There is a significant reduction in IR violations in SFU.

IEL v ZI v SFU

Draws = average number of trades = 40.



IEL's CDF for efficiency is closer to the SFU CDF than ZI's.

ZI's CDF for price is closer to the SFU CDF than IEL's.

– Mixing NI with IEL does not improve things.

Summary

Goal: Provide a model of individual behavior that explains the data from the first periods of experimental CDAs, without calibration.

Today: Compared two theories (IEL and ZI) to two data sets (MobLab and SFU).

- IEL +30%NI explains MobLab better than ZI
- IEL explains SFU efficiencies much better than ZI.
- ZI explains SFU prices a little better than IEL.

Future Work on IEL

An agent's decision about when to order should be endogenous.
– Gjerstad-Dickhaut (2003) a possibility but is complicated.
– van de Leur-Anufriev (2018) is another.

Modifications to IEL to explain the SFU price CDF better.
– SFU prices more dispersed than ZI or IEL.

Interactions between IEL and other computer and human agents.
– If IEL interacts with humans, behavior may be different.

IEL can look further back to evaluate possible strategies.
– van de Leur-Anufriev (2018) use multi-period learning in IEL

Adapt IEL to multiple commodities.
– Build on Asparouhova, Bossaerts, and Ledyard (2020)